NATIONAL SENIOR CERTIFICATE EXAMINATION
NOVEMBER 2013

MATHEMATICS: PAPER II

MARKING GUIDELINES

Time: 3 hours 150 marks

These marking guidelines are prepared for use by examiners and sub-examiners, all of whom are required to attend a standardisation meeting to ensure that the guidelines are consistently interpreted and applied in the marking of candidates' scripts.

The IEB will not enter into any discussions or correspondence about any marking guidelines. It is acknowledged that there may be different views about some matters of emphasis or detail in the guidelines. It is also recognised that, without the benefit of attendance at a standardisation meeting, there may be different interpretations of the application of the marking guidelines.

QUESTION 1

(a) (1) (i) $\frac{51\ 770\ 561}{9}$ (divide by 9)

$= 5\ 752\ 285$ (2)

(ii) $\frac{12\ 272\ 263}{51\ 770\ 561} = 23{,}7\%$ (1)

(iii) $\frac{12\ 272\ 263}{18\ 178} \simeq$ 675 people/km^2(accept 676 and 675,1) (1)

(b) (1) 2% of 12 272 263 $\simeq$ 24 5445(accept +/- 1) (1)

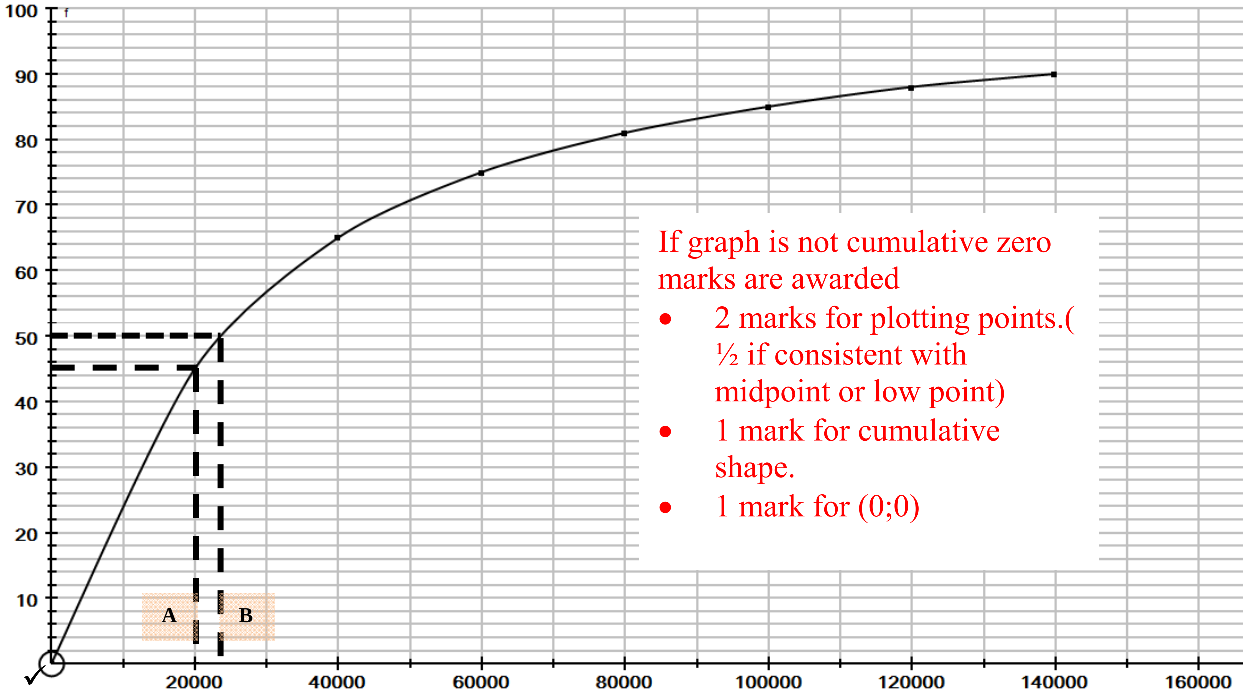
(2) 10% (1)

(3) (i)

Category	**Approximate Percentage**	**Cumulative Percentage Frequency**
$R0 \leq x < R20\,000$	45%	45%
$R20\,000 \leq x < R40\,000$	20%	65%
$R40\,000 \leq x < R60\,000$	10%	75%
$R60\,000 \leq x < R80\,000$	6%	81%
$R80\ 000 \leq x < R100\,000$	4%	85%
$R100\ 000 \leq x < R120\,000$	3%	88%
$R120\ 000 \leq x < R140\,000$	2%	90%

(The second mark is awarded, for 90%, given that all other entries are correct) (2)

(3) (ii)



(4)

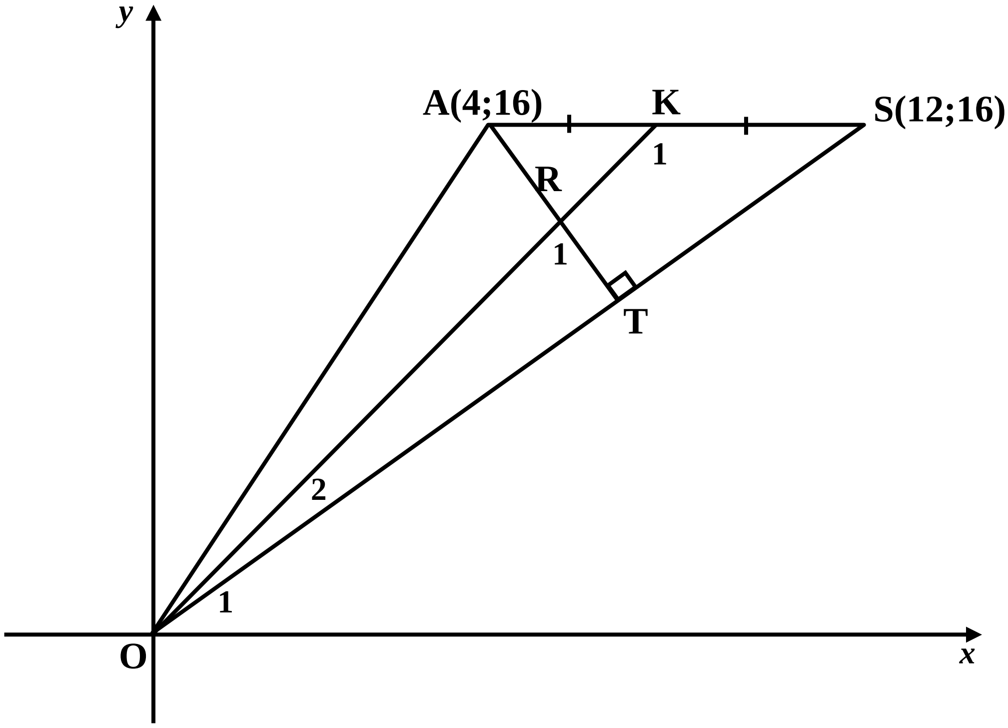
(4) (i) Indicated by A (1)

(ii) Indicated by B (1)

[14]

QUESTION 2

(a)



$$K\left(\frac{4+12}{2};\frac{16+16}{2}\right)$$

(1) $K(8;16)$

$$m_{OK}=\frac{16-0}{8-0}\checkmark=2$$

$\therefore \quad y=2x$ (5)

(2) $m_{OS}=\dfrac{16-0}{12-0}=\dfrac{4}{3}$

$\therefore m_{AT}=-\dfrac{3}{4}$

$y-16=-\dfrac{3}{4}(x-4)$

$\therefore \quad y=-\dfrac{3}{4}x+19$ (4)

(3) (i) $\tan\hat{O}_1=\dfrac{16}{12}$

$\therefore \quad \hat{O}_1=53,1^\circ$ (2)

(ii) $\tan\left(\hat{O}_{1+2}\right)=2$

$\hat{O}_{1+2}=63,4^\circ$

Therefore, $\hat{O}_2=10,3^\circ$ (3)

(iii) AS // x-axis

$\hat{R}_1 = 90° - 10,3°$ $\hat{K}_1 = 180° - (53,1° + 10,3°) = 116,6°$

$\therefore R_1 = 79,7°$ (3)

(b) Centre of the circle is $(-2;-2)$

Therefore, equation of circle is $(x+2)^2 + (y+2)^2 = r^2$

Substituting $B(-8;4)$, we get $(-8+2)^2 + (4+2)^2 = r^2$

$r^2 = 72$

Therefore equation of circle is $(x+2)^2 + (y+2)^2 = 72$

OR

$r = \frac{1}{2}BD \therefore r^2 = \frac{1}{4}BD^2 = \frac{1}{4}\left[12^2 + 12^2\right] = 72$ (4)

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QUESTION 3

(a) (1) B (1)

(2) E (1)

(b) (1) D (1)

(2) C (1)

(c) (1) $(x;y) \rightarrow \left(\frac{1}{3}x;\frac{1}{3}y\right)$ (description accepted) (1)

(2) (1)

(d) (1) $(x;y) \rightarrow (-x;-y)$ (2)

(2) $(x;y) \rightarrow (x\cos 210° + y\sin 210°; y\cos 210° - x\sin 210°)$ for 150 or 210

$(x;y) \rightarrow \left(-\frac{\sqrt{3}}{2}x - \frac{1}{2}y; -\frac{\sqrt{3}}{2}y + \frac{1}{2}x\right)$ (4)

[12]

QUESTION 4

(a) (1) $\cos\hat{G} = 0{,}726$

$\hat{G} = 360 - 43{,}4 = 316{,}6°$ (2)

(2) $\tan\left(\frac{2}{3}\hat{G} + 100°\right) = -1{,}148$ (accept -1,1) (1)

(b)

$$\frac{\sin(180° - A)}{\cos(90° + A) + \sin(360° - A)}$$

$$= \frac{\sin A}{-\sin A - \sin A}$$

$$= -\frac{1}{2}$$ (4)

(c) (1) $\frac{k}{8} = \frac{1}{4}$

$\therefore \quad k = 2$ (2)

(2) $OT^2 = 8^2 + 2^2 = 68$

Therefore,

$$\sin\beta = \frac{k}{OT}$$

$$= \frac{2}{\sqrt{68}} \text{ or } \frac{1}{\sqrt{17}}$$ (3)

(d)

$$\frac{\cos(45° - \theta)}{\cos 45°.\cos\theta} - \tan\theta$$

$$= \frac{\cos 45°.\cos\theta + \sin 45°.\sin\theta}{\frac{\sqrt{2}}{2}.\cos\theta} - \frac{\sin\theta}{\cos\theta}$$

$$= \frac{\frac{\sqrt{2}}{2}.\cos\theta + \frac{\sqrt{2}}{2}.\sin\theta}{\frac{\sqrt{2}}{2}.\cos\theta} - \frac{\sin\theta}{\cos\theta}$$

$$= \frac{\cos\theta + \sin\theta}{\cos\theta} - \frac{\sin\theta}{\cos\theta}$$

$$= 1$$

Expansion Incorrect: Max of 2.

OR

$$\frac{\cos 45°.\cos\theta + \sin 45°.\sin\theta}{\cos 45°.\cos\theta} - \tan\theta$$

$$= \frac{\cos\theta + \sin\theta}{\cos\theta} - \frac{\sin\theta}{\cos\theta} \ldots$$

$$= \frac{\cos\theta}{\cos\theta} \quad (\text{since } \sin 45° = \cos 45°)$$

$$= 1$$

OR

$$\frac{\cos 45.\cos\theta + \sin 45°\sin\theta}{\cos 45°\cos\theta} - \tan\theta$$

$$= 1 + \tan 45.\tan\theta - \tan\theta$$

=1+1.tanϴ-tanϴ

$$= 1$$ (5)

[17]

QUESTION 5

(a) (1) 120° (1)

(2) a = 3 (1)

(3) 60° (1)

(4) b = 3 (1)

(b) (1) $P(60°;4\tan 60°) = P(60°;4\sqrt{3})$ (2)

(2) $2\sin x = 1,5\cos x$
$\tan x = 0,75$
$x = 36,87°$

$Q(36,87°;1,20)$ (4)

(3) Many possible answers, e.g. (15°; 1) (2)

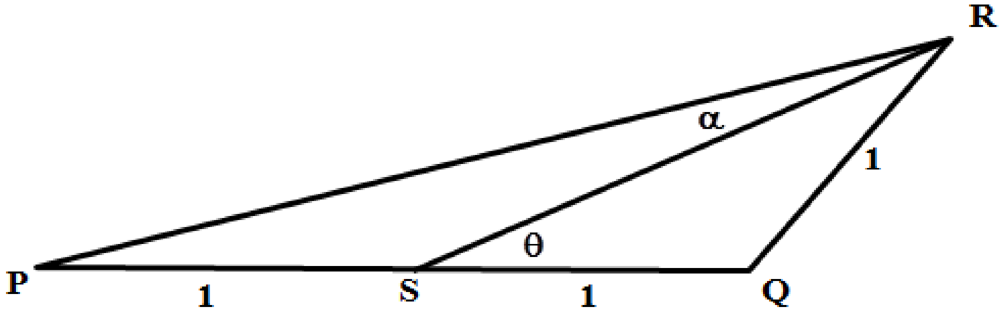
[12]

76 marks

SECTION B

QUESTION 6

(a)



(1) In ΔPSR,

$\hat{P} + \alpha = \theta \therefore \ \hat{P} = \theta - \alpha$ (1)

(2) $\dfrac{SR}{\sin(\theta-\alpha)} = \dfrac{PS}{\sin\alpha} \quad \therefore \dfrac{2\cos\theta}{\sin(\theta-\alpha)} = \dfrac{1}{\sin\alpha}$

$\dfrac{SR}{\sin(180-2\theta)} = \dfrac{1}{\sin\theta}$

$SR = \dfrac{\sin 2\theta}{\sin\theta}$

$SR = \dfrac{2\sin\theta\cos\theta}{\sin\theta}$

$SR = 2\cos\theta$

$\sin(\theta-\alpha) = 2\cos\theta.\sin\alpha$

$\sin\theta.\cos\alpha - \sin\alpha.\cos\theta = 2\cos\theta.\sin\alpha$

$\sin\theta.\cos\alpha = 3\cos\theta.\sin\alpha$

$\therefore \dfrac{\sin\theta}{\cos\theta} = \dfrac{3\sin\alpha}{\cos\alpha}$

$\therefore \tan\theta = 3\tan\alpha$

OR

$S\hat{R}Q = \theta$

In Δ PRQ

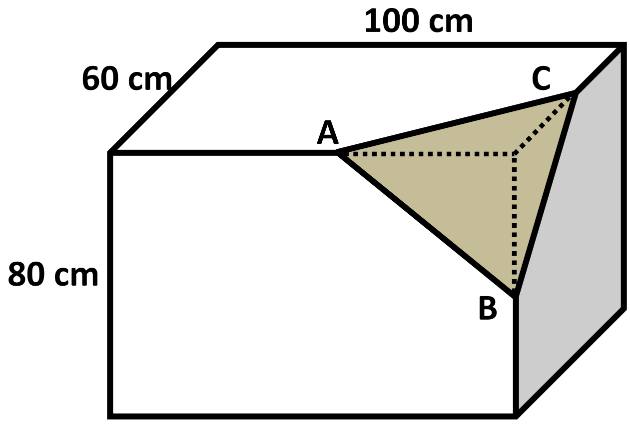
$\dfrac{2}{\sin(\alpha+\theta)} = \dfrac{1}{\sin(\theta-\alpha)}$

$\therefore 2[\sin\theta\cos\alpha - \sin\alpha\cos\theta] = \sin\alpha\cos\theta + \sin\theta\cos\alpha$

$\therefore \sin\theta\cos\alpha = 3\sin\alpha\cos\theta$

$\therefore \tan\theta = 3\tan\alpha$ (6)

(b)



$AC^2 = 50^2 + 30^2 = 3400$

$AB^2 = 50^2 + 40^2 = 4100$

$BC^2 = 40^2 + 30^2 = 2500$

In ΔABC, $4100 = 3400 + 2500 - 2 \times \sqrt{3400} \times \sqrt{2500} \times \cos C$

$\therefore \quad \cos C = 0{,}308697...$

$\therefore \quad C = 72{,}0192495°$

The area of $\Delta ABC = 0{,}5 \times \sqrt{3400} \times 50 \times \sin 72{,}0192...° = 1386{,}5 \text{ cm}^2$.

[Please note that the students can calculate cos A or cos B instead of cos C.]

(7)

[14]

QUESTION 7

(a) $\text{RHS} = \dfrac{\cos 2A - \cos 4A}{\sin 4A - \sin 2A} = \dfrac{\cos(3A - A) - \cos(3A + A)}{\sin(3A + A) - \sin(3A - A)}$

$\text{RHS} = \dfrac{(\cos 3A.\cos A + \sin 3A.\sin A) - (\cos 3A.\cos A - \sin 3A.\sin A)}{(\sin 3A.\cos A + \sin A.\cos 3A) - (\sin 3A.\cos A - \sin A.\cos 3A)}$

$\text{RHS} = \dfrac{(2\sin 3A.\sin A)}{(2\sin A.\cos 3A)} = \tan 3A = \text{LHS}$

Therefore, $\tan 3A = \dfrac{\cos 2A - \cos 4A}{\sin 4A - \sin 2A}$ (7)

(b) $\sin 4A = \sin 2A$

$\therefore 4A = 2A + K.360°; \quad K \in Z$ or $4A = 180° - 2A + K.360°; K \in Z$

$2A = K.360°$ $\quad \therefore 6A = 180° + K.360°$

$A = K.180°$ $\quad \therefore \quad A = 30° + K.60°$

(7)

OR

$\sin 4A - \sin 2A = 0$

$2\sin 2A \cos 2A - \sin 2A = 0$

$\sin 2A(2\cos 2A - 1) = 0$

Sin2A=0 or 2cos2A=1

$2A = 0 + 180k$ $\quad 2A = \pm 60 + 360k$ $\quad k \in Z$

$A = 90k$ or $A = \pm 30 + 180k$;

OR

$\sin(3A + A) = \sin(3A - A)$

$\sin 3A.\cos A + \sin A.\cos 3A = \sin 3A.\cos A - \sin A.\cos 3A$

$2\sin A.\cos 3A = 0$

$\sin A = 0 \quad or \quad \cos 3A = 0$

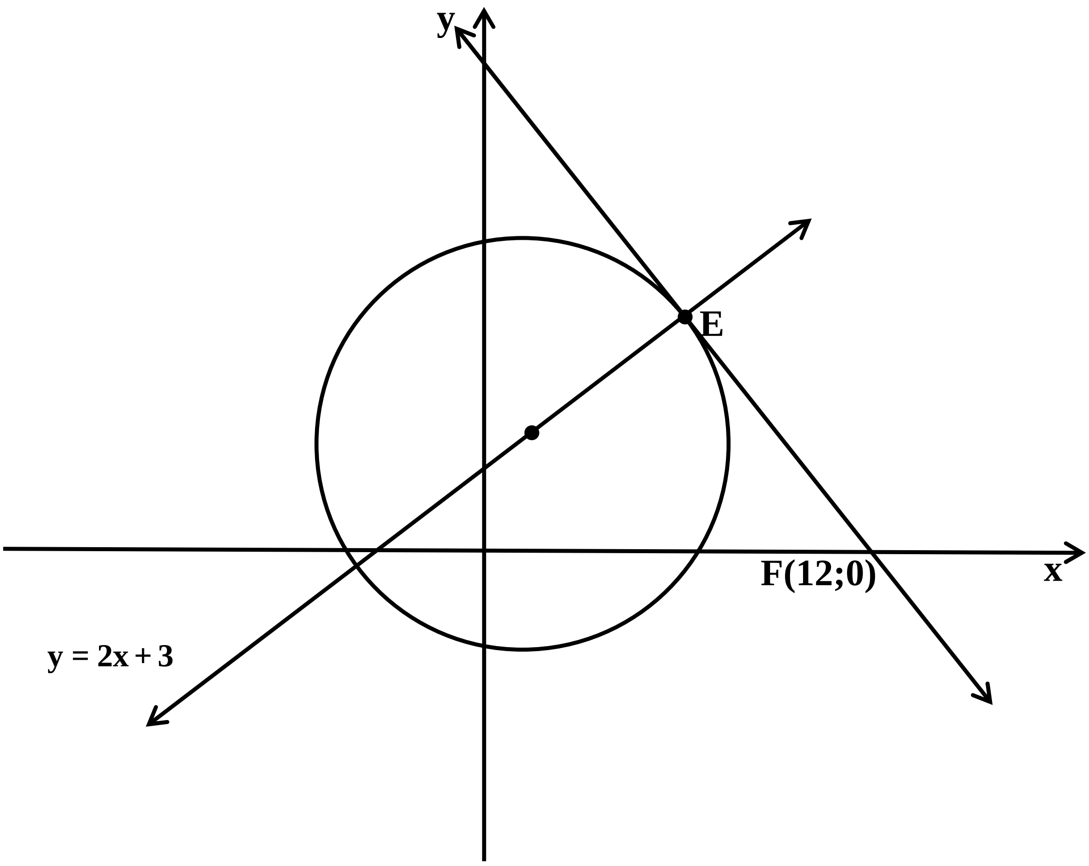
$A = 0 + 180k \quad or \quad 3A = 90 + 180k$

$A = 30 + 60k; \quad k \in Z$

[14]

QUESTION 8

(a)



$m_{tan} = -\frac{1}{2}$ since tangent at E $\perp$ $y = 2x + 3$

$\therefore \quad y - 0 = -\frac{1}{2}(x - 12)$

$\therefore \quad y = -\frac{1}{2}x + 6$

For E, $\quad -\frac{1}{2}x + 6 = 2x + 3$

$\therefore \quad -x + 12 = 4x + 6$

$\therefore \quad -5x = -6$

$\therefore \quad x = \frac{6}{5}$

$\therefore \quad y = 2 \times \frac{6}{5} + 3 = \frac{27}{5} \qquad \therefore \quad E\left(\frac{6}{5}; \frac{27}{5}\right)$ (6)

(b)

$B(2p;-p)$

$OB^2 = (2p)^2 + (-p)^2 = 5p^2$

$\therefore \quad \sqrt{5}p = \sqrt{20} + \sqrt{45}$

$\therefore \quad \sqrt{5}p = 2\sqrt{5} + 3\sqrt{5}$

$\therefore \quad p = 5$ (accept p = -5) (5)

(a) (1) $x^2 + y^2 + 4x.\cos\theta + 8y.\sin\theta + 3 = 0$

$x^2 + 4x.\cos\theta + 4\cos^2\theta + y^2 + 8y\sin\theta + 16\sin^2\theta = -3 + 16\sin^2\theta + 4\cos^2\theta$

$(x + 2\cos\theta)^2 - 4\cos^2\theta + (y + 4\sin\theta)^2 - 16\sin^2\theta + 3 = 0$

$(x + 2\cos\theta)^2 + (y + 4\sin\theta)^2 = 4\cos^2\theta + 16\sin^2\theta - 3$

$r^2 = 4(1 - \sin^2\theta) + 16\sin^2\theta - 3$

$r^2 = 1 + 12\sin^2\theta \le 13$ for all values of θ

$\therefore r \le \sqrt{13}$ (5)

[16]

QUESTION 9

(a) $\frac{3+x+y+12+10}{5}=8$

$x+y=15$ ....A

squaring difference from mean

$\frac{(-5)^2+(x-8)^2+(y-8)^2+2^2+4^2}{5}=10$

$(x-8)^2+(y-8)^2=5$ ....B

Substituting A into B, we get

$(x-8)^2+(15-x-8)^2=5$

$x^2-16x+64+x^2-14x+49=5$

$2x^2-30x+108=0$

$x^2-15x+54=0$

$(x-6)(x-9)=0$

$x=6$ or $x=9$

$y=9$ $\quad y=6$

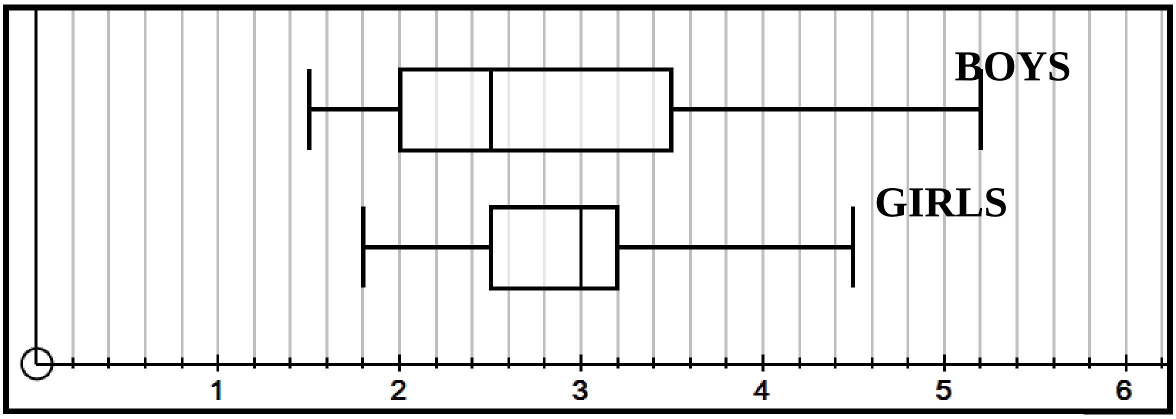
Many students have used the method of trial and error together with their calculators:

2 marks: any pair of values that add to 15

8 marks: x = 6 and y = 9

(8)

(b)



(1) The slowest individual was a boy with a time of 5,2 minutes. That means all the girls were finished before all the boys were finished.
75% of the girls were finished with more than 25% of the boys still needing to finish. (2)

(2) Bottom 25% of the girls faster than bottom 25% of the boys, etc.
The fastest individual was a boy with a time of 1,5 minutes.
Half of the boys were finished when only one quarter of the girls were finished. (2)

[12]

QUESTION 10

(a) (1) $(x; y) \rightarrow (x - 6; y + 6)$ (2)

(2) $(x; y) \rightarrow (y; x)$ (2)

(b) (1) $S(-6; 0) \quad Q(-6; -12)$

$\therefore S'(-6\cos\theta\ ; 6\sin\theta) \qquad Q'(-6\cos\theta - 12\sin\theta; -12\cos\theta + 6\sin\theta)$ (6)

(2) $m_{S'Q'} = \dfrac{-12\cos\theta}{-12\sin\theta} = \dfrac{\cos\theta}{\sin\theta}$ **OR** $m_{S'Q'} = \tan(90-\theta)$

$= \dfrac{\sin(90-\theta)}{\cos(90-\theta)}$

$= \dfrac{\cos\theta}{\sin\theta}$

$\therefore$ eqn of $S'Q'$: $y - 6\sin\theta = \dfrac{\cos\theta}{\sin\theta}(x + 6\cos\theta)$

$\therefore y = \dfrac{x.\cos\theta}{\sin\theta} + \dfrac{6\cos^2\theta}{\sin\theta} + \dfrac{6\sin^2\theta}{\sin\theta}$

$\therefore y = \dfrac{x.\cos\theta + 6}{\sin\theta}$ (3)

(3) Area OBCS'

$= 2 \times$ area OBC

$= 2 \times \dfrac{1}{2} \times 6 \times BC$

$= 6$ BC

Since $y_c = -6$

$-6 = \dfrac{x_c\cos\theta + 6}{\sin\theta}$

$\therefore -6\sin\theta = x_c\cos\theta + 6$

$\therefore x_c\cos\theta = -6\sin\theta - 6$

$x_c = -6\tan\theta - \dfrac{6}{\cos\theta}$

$\therefore \text{Area} = \left[\dfrac{1}{2} \times \left(6 \times \left(6\tan\theta + \dfrac{6}{\cos\theta}\right)\right)\right] \times 2$

$= 36\tan\theta + \dfrac{36}{\cos\theta}$ (5)

[18]

74 marks

Total: 150 marks